1:30 - 3:30
32-155

MIT ID# (last four digits) SOLUTIONS

Unified Quiz MS5

December 18, 2007

M - PORTION

EXAM SCORING

#1M (50%)	
#2M (50%)	
FINAL SCORE	

Unified Quiz MS5 - Materials and Structures Portion
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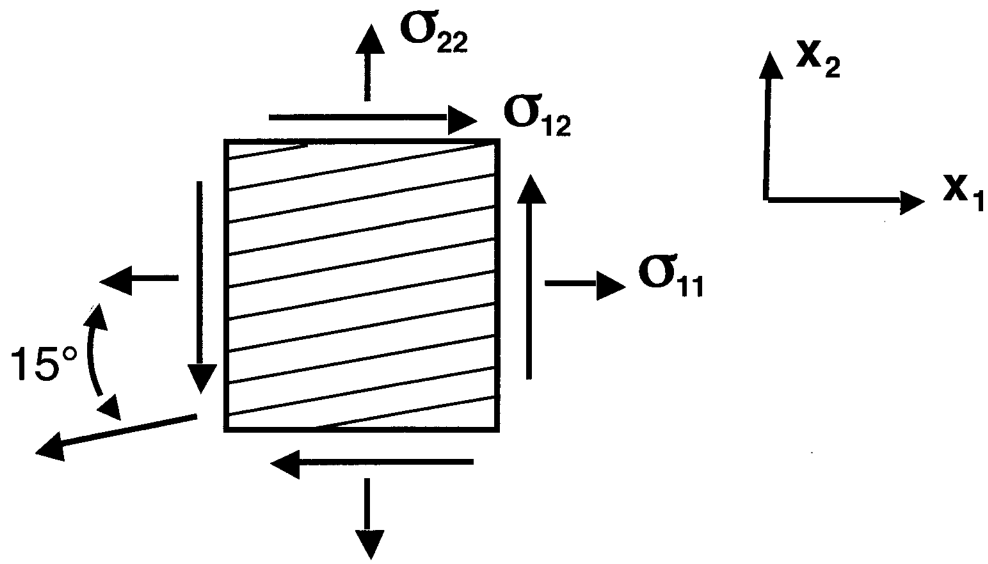
PROBLEM #1M (50%)

A unidirectional composite material has a general loading applied resulting in stresses of σ_{11}= 30 MPa, σ_{22}= 15 MPa, and σ_{12}= 10 MPa. These stresses are measured in an axis system at an angle of 15° to all the fibers of the material, as indicated in the accompanying figure.



Determine the principal in-plane stresses and their direction or **clearly** indicate why they cannot be determined.

First important point is that the fiber angle orientation plays no role here. The principal stresses are a function only of the applied stress state.

For an in-plane stress system, the principal stresses are the roots of the equation:

$$\tau^2 - \tau(\sigma_{11} + \sigma_{22}) + (\sigma_{11}\sigma_{22} - \sigma_{12}^2) = 0$$

here we have: $\sigma_{11} = 30\,MPa$
$\sigma_{22} = 15\,MPa$
$\sigma_{12} = 10\,MPa$

So: $$\tau^2 - \tau(45\,MPa) + (450\,MPa^2 - 100\,MPa^2) = 0$$

PROBLEM #1M (continued)

giving: $\tau^2 - \tau(45\,MPa) + 350\,[MPa]^2 = 0$

Solve via the quadratic formula: $\tau = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

here: $a = 1$
$b = -45\,MPa$
$c = 350\,[MPa]^2$

$$\Rightarrow \tau = \frac{45\,MPa \pm \sqrt{(45)^2 - 4(1)(350)}\ MPa}{2}$$

$$= \frac{45\,MPa \pm \sqrt{2025 - 1400}\ MPa}{2}$$

$$= \frac{45\,MPa \pm \sqrt{625}\ MPa}{2} = \frac{45\,MPa \pm 25\,MPa}{2}$$

$$\Rightarrow \tau = \frac{70}{2}\,MPa,\ \frac{20}{2}\,MPa$$

So: $\boxed{\text{Principal stresses: } \sigma_I, \sigma_{II} = 35\,MPa,\ 10\,MPa}$

Note: Check via Σ extensional = constant = 45 MPa ✓

Get the direction via:

$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{2\sigma_{12}}{\sigma_{11} - \sigma_{22}}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{20}{15}\right) = \frac{1}{2}\tan^{-1}\left(\frac{4}{3}\right)$$

$$= \frac{1}{2}(53.1^\circ)$$

$\Rightarrow \boxed{\text{Principal direction: } \theta_p = 26.6^\circ}$

from x_1 - axis

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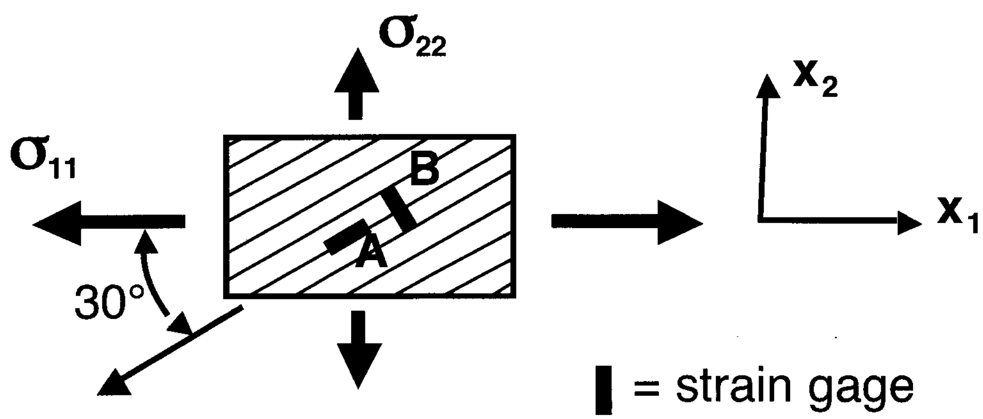
PROBLEM #2M (50%)

A unidirectional composite ply is oriented at an angle of 30° to the loading axes in which a biaxial stress is applied as shown in the accompanying figure. A stress of 30 ksi is applied along the x_1-axis and a stress of -10 ksi perpendicular to this along the x_2-axis. In addition, there are two strain gages placed on the surface of the ply: one (Gage A) along the fibers, and one (Gage B) perpendicular to the fibers. These are also shown in the accompanying figure. The strain gages read:

Gage A = 1000 μstrain   Gage B = -300 μstrain



Can any of the engineering constants of this unidirectional composite material be determined? If so, explain why and do so. If not, explain why not. Equations can be utilized in the explanation in either case.

→ The orthotropic stress-strain equations are (for in-plane loading):

$$\epsilon_{11} = \frac{1}{E_1}\sigma_{11} - \frac{\nu_{12}}{E_1}\sigma_{22}$$

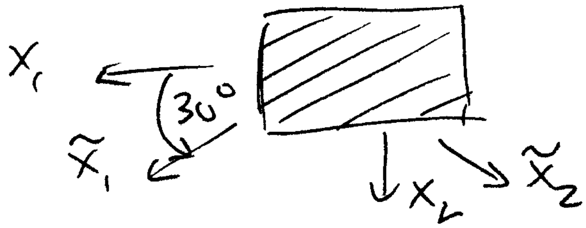
$$\epsilon_{22} = -\frac{\nu_{21}}{E_2}\sigma_{11} - \frac{1}{E_2}\sigma_{22}$$

$$\epsilon_{12} = \frac{1}{2G_{12}}\sigma_{12}$$

this describes the response in the axis system aligned with the fibers.

→ It is therefore necessary to rotate the stress state into the fiber axis system (the material principal axes):

PROBLEM #2M (continued)



$$\tilde{\sigma}_{11} = \sigma_{11}\cos^2\theta + \sigma_{22}\sin^2\theta + 2\cos\theta\sin\theta\,\sigma_{12}$$

$$\tilde{\sigma}_{22} = \sigma_{11}\sin^2\theta + \sigma_{22}\cos^2\theta - 2\cos\theta\sin\theta\,\sigma_{12}$$

$$\tilde{\sigma}_{12} = -\sin\theta\cos\theta\,\sigma_{11} + \sin\theta\cos\theta\,\sigma_{22} + (\cos^2\theta - \sin^2\theta)\,\sigma_{12}$$

Here: $\sigma_{11} = 30\text{ksi}$

$\sigma_{22} = -10\text{ksi}$

$\sigma_{12} = 0$

$\cos^2\theta = 3/4 \qquad \sin\theta\cos\theta = 0.433$

$\sin^2\theta = 1/4$

$$\Rightarrow \tilde{\sigma}_{11} = (30\text{ksi})(3/4) + (-10\text{ksi})(1/4) = 22.5\text{ksi} - 2.5\text{ksi}$$

$$\Rightarrow \tilde{\sigma}_{11} = 20\text{ksi}$$

$$\tilde{\sigma}_{22} = (30\text{ksi})(1/4) + (-10\text{ksi})(3/4) = 7.5\text{ksi} - 7.5\text{ksi} = 0$$

$$\Rightarrow \tilde{\sigma}_{22} = 0\text{ksi}$$

$$\tilde{\sigma}_{12} = -(0.433)(30\text{ksi}) + (0.433)(-10\text{ksi}) = -17.3\text{ksi}$$

$$\Rightarrow \tilde{\sigma}_{12} = -17.3\text{ksi}$$

→ So we have a stress state in the fiber axis system of:

$\tilde{\sigma}_{11} = 20\text{ksi}$

$\tilde{\sigma}_{22} = 0\text{ksi}$

$\tilde{\sigma}_{12} = -17.3\text{ksi}$

PROBLEM #2M (continued)

→ we have strain measured in the ~ system:

$\tilde{\epsilon}_{11} = 1000\,\mu\text{strain}$

$\tilde{\epsilon}_{22} = -300\,\mu\text{strain}$

→ use the orthotropic stress-strain equations representing behavior in the fiber axes (material principal axes):

$$\epsilon_{11} = 1000\,\mu\text{strain} = \frac{1}{E_1}\sigma_{11} - \frac{\nu_{12}}{E_1}\sigma_{22}$$

$$= \frac{1}{E_1}(20\,\text{ksi})$$

$$\Rightarrow E_1 = \frac{20\times10^3\,\text{psi}}{1000\times10^{-6}} = 20\times10^6\,\text{psi}$$

$$\Rightarrow \boxed{E_1 = 20\,\text{Msi}}$$

Modulus along fibers

and:

$$\epsilon_{22} = -300\,\mu\text{strain} = -\frac{\nu_{21}}{E_2}\sigma_{11} - \frac{1}{E_2}\sigma_{22}$$

Recall reciprocity: $\nu_{21}E_1 = \nu_{12}E_2$

$$\Rightarrow \frac{\nu_{21}}{E_2} = \frac{\nu_{12}}{E_1}$$

So: $$-300\times10^{-6} = \frac{\nu_{12}}{20\times10^6\,\text{psi}}(20\times10^3\,\text{psi})$$

$$\Rightarrow \nu_{12} = -(-300\times10^{-6})(10^3) = 0.30$$

$$\Rightarrow \boxed{\nu_{12} = 0.30}$$

major Poisson's ratio